Centre Number			Candidate Number		
Surname					
Other Names					
Candidate Signature					



General Certificate of Education Advanced Level Examination June 2011

Mathematics

MPC3

Unit Pure Core 3

Monday 13 June 2011 9.00 am to 10.30 am

For this paper you must have:

• the blue AQA booklet of formulae and statistical tables. You may use a graphics calculator.

Time allowed

1 hour 30 minutes

Instructions

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer all questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer the questions in the spaces provided. Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

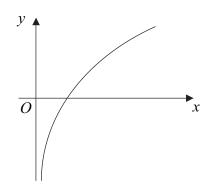
Advice

 Unless stated otherwise, you may quote formulae, without proof, from the booklet.

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Question	Mark
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TOTAL	

Answer all questions in the spaces provided.

1 The diagram shows the curve with equation $y = \ln(6x)$.



(a) State the x-coordinate of the point of intersection of the curve with the x-axis. (1 mark)

(b) Find $\frac{dy}{dx}$. (2 marks)

Use Simpson's rule with 6 strips (7 ordinates) to find an estimate for $\int_{1}^{7} \ln(6x) dx$, giving your answer to three significant figures. (4 marks)

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- **2 (a) (i)** Find $\frac{dy}{dx}$ when $y = xe^{2x}$. (3 marks)
 - (ii) Find an equation of the tangent to the curve $y = xe^{2x}$ at the point $(1, e^2)$. (2 marks)
 - **(b)** Given that $y = \frac{2 \sin 3x}{1 + \cos 3x}$, use the quotient rule to show that

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{k}{1 + \cos 3x}$$

where k is an integer.

(4 marks)

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- The curve $y = \cos^{-1}(2x 1)$ intersects the curve $y = e^x$ at a single point where $x = \alpha$.
 - (a) Show that α lies between 0.4 and 0.5. (2 marks)
 - (b) Show that the equation $\cos^{-1}(2x-1) = e^x$ can be written as $x = \frac{1}{2} + \frac{1}{2}\cos(e^x)$. (1 mark)
 - Use the iteration $x_{n+1} = \frac{1}{2} + \frac{1}{2}\cos(e^{x_n})$ with $x_1 = 0.4$ to find the values of x_2 and x_3 , giving your answers to three decimal places. (2 marks)

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- **4 (a) (i)** Solve the equation $\csc \theta = -4$ for $0^{\circ} < \theta < 360^{\circ}$, giving your answers to the nearest 0.1°. (2 marks)
 - (ii) Solve the equation

$$2\cot^2(2x+30^\circ) = 2 - 7\csc(2x+30^\circ)$$

for $0^{\circ} < x < 180^{\circ}$, giving your answers to the nearest 0.1°. (6 marks)

(b) Describe a sequence of two geometrical transformations that maps the graph of $y = \csc x$ onto the graph of $y = \csc(2x + 30^\circ)$. (4 marks)

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5	The functions f and g are defined with their respective domains by	
	$f(x) = x^2$ for all real values of x	
	$g(x) = \frac{1}{2x+1}$ for real values of x , $x \neq -0.5$	
(a	Explain why f does not have an inverse.	(1 mark)
(b	The inverse of g is g^{-1} . Find $g^{-1}(x)$.	(3 marks)
(с	State the range of g^{-1} .	(1 mark)
(d	Solve the equation $fg(x) = g(x)$.	(3 marks)
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12

6 (a)	Given that $3 \ln x = 4$, find the exact value of x.	(1 mark)
(b)	ΠX	
	answers for x in an exact form.	(5 marks)
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7 (a)	On separate diagrams:	
	(i)	sketch the curve with equation $y = 3x + 3 $;	(2 marks)
	(ii)	sketch the curve with equation $y = x^2 - 1 $.	(3 marks)
(b) (i)	Solve the equation $ 3x + 3 = x^2 - 1 $.	(5 marks)
	(ii)	Hence solve the inequality $ 3x + 3 < x^2 - 1 $.	(2 marks)
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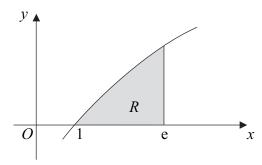
8	Use the substitution $u = 1 + 2 \tan x$ to find
	$\int \frac{1}{(1+2\tan x)^2\cos^2 x} \mathrm{d}x \tag{5 marks}$
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- **9 (a)** Use integration by parts to find $\int x \ln x \, dx$. (3 marks)
 - **(b)** Given that $y = (\ln x)^2$, find $\frac{dy}{dx}$. (2 marks)
 - (c) The diagram shows part of the curve with equation $y = \sqrt{x} \ln x$.



The shaded region R is bounded by the curve $y = \sqrt{x} \ln x$, the line x = e and the x-axis from x = 1 to x = e.

Find the volume of the solid generated when the region R is rotated through 360° about the x-axis, giving your answer in an exact form. (6 marks)

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